Fórmulas de reducción trigonométricas

$$I_n = \int sen^n u \, du = -\frac{1}{n} sen^{(n-1)} u \cos u + \frac{n-1}{n} I_{n-2}$$

$$I_n = \int \cos^n u \, du = \frac{1}{n} \cos^{(n-1)} u \; sen\, u + \frac{n-1}{n} I_{n-2}$$

$$I_n = \int \sec^n u \, du = \frac{1}{n-1} \sec^{(n-2)} u \tan u + \frac{n-2}{n-1} I_{n-2}$$

$$I_n = \int \csc^n u \, du = -\frac{1}{n-1} \csc^{(n-2)} u \cot u + \frac{n-2}{n-1} I_{n-2}$$

$$I_n = \int \tan^n u \, du = \frac{1}{n-1} \tan^{n-1} u - I_{n-2}$$

Ejemplo: $\underset{n=2}{I_2} = \int sen^2\theta \, d\theta = -\frac{1}{2} sen^{(1)}\theta \cos\theta + \frac{1}{2}\overset{n-2=0}{\int d\theta} = -\frac{1}{2} sen\,\theta \cos\theta + \frac{\theta}{2} + C$

$$= \frac{\theta}{2} - \frac{sen\,\theta\cos\theta}{2} + C = \frac{2\theta - 2\,sen\,\theta\cos\theta}{4} + C = \frac{2\theta - sen(2\theta)}{4} + C$$

Fórmulas de cuadrados trigonométricos

$$\int sen^2 u \, du = \frac{2u - sen(2u)}{4} + C \qquad \int \cos^2 u \, du = \frac{\cos u \; sen\, u + u}{2} + C$$

$$\int \tan^2 u \, du = \tan u - u + C \qquad \int \cot^2 u \, du = -\cot u - u + C$$

$$\int \sec^2 u \, du = \tan u + C \qquad \int \csc^2 u \, du = -\cot u + C$$

Identidades Hiperbólicas

Las funciones hiperbólicas son análogas a las trigonométricas, pero se basan en la curva hipérbola en lugar del círculo.

$$senh\,x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2} \qquad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

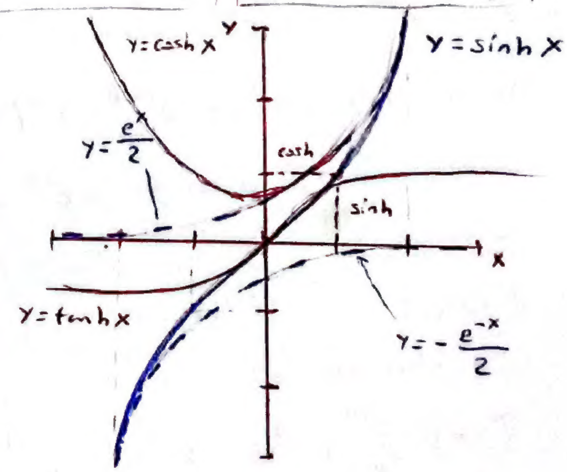
$$\coth x = \frac{1}{\tanh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}} \qquad sech\,x = \frac{1}{\cosh x} \qquad csch\,x = \frac{1}{senh\,x}$$

$$\cosh^2 x - senh^2 x = 1 \qquad senh\,2x = 2\,senh\,x\cosh x \qquad \cosh 2x = \cosh^2 x + senh^2 x$$

$$\cosh^2 x = \frac{\cosh 2x + 1}{2} \qquad senh^2 x = \frac{\cosh 2x - 1}{2} \qquad \tanh^2 x = 1 - sech^2 x \qquad \coth^2 x = 1 + csch^2 x$$

Derivadas e integrales (7.7 p.428)

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x) dx$
$senh\,x$	$\cosh x$	$\cosh x + C$
$\cosh x$	$senh\,x$	$senh\,x + C$
$\tanh x$	$sech^2 x$	$\ln(\cosh x) + C$
$\coth x$	$-csch^2 x$	$\ln(senh\,\lvert x\rvert) + C$
$sech\,x$	$-sech\,x \tanh x$	$\arctan(senh\,x) + C$
$csch\,x$	$-csch\,x \coth x$	$\ln\left(\tanh\frac{\lvert x\rvert}{2}\right) + C$
$senh^2 x$	$2\cosh x \sinh x$	$\dfrac{\frac{e^{2x}}{2} - \frac{e^{-2x}}{2} - 2x}{4} + C$
$\cos^2 x$	$2\cosh x \sinh x$	$\dfrac{\frac{e^{2x}}{2} - \frac{e^{-2x}}{2} + 2x}{4} + C$
$\tan^2 x$	$2\,sech^2 x \tanh x$	$x - \frac{2}{e^{2x} + 1} + C$



Número e: $e \simeq \left(1+\frac{1}{n}\right)^n$ Ejemplo: $1'001^{1000} = \left(1+\frac{1}{1000}\right)^{1000} \approx e$

$a^n = e^{n \ln a} \quad \forall a, n \in \mathbb{R}$

Identidades trigonométricas

- $\operatorname{sen}^2 x + \cos^2 x = 1$ - $\sec^2 x - \tan^2 x = 1$ - $\csc^2 x - \cot^2 x = 1$

- $\operatorname{sen} 2x = 2 \operatorname{sen} x \cos x$ - $\cos 2x = \cos^2 x - \operatorname{sen}^2 x = 2\cos^2 x - 1 = 1 - 2\operatorname{sen}^2 x$
- $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

- $\operatorname{sen}^2 x = \dfrac{1-\cos 2x}{2}$ - $\cos^2 x = \dfrac{1+\cos 2x}{2}$ - $\tan^2 x = \dfrac{1-\cos 2x}{1+\cos 2x}$

- $\operatorname{sen}(u \pm v) = \operatorname{sen} u \cos v \pm \cos u \operatorname{sen} v$ - $\cos(u \pm v) = \cos u \cos v \mp \operatorname{sen} u \operatorname{sen} v$

- $\tan(u \pm v) = \dfrac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$

Integrales

$\displaystyle\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$ arctan

$\displaystyle\int \frac{dx}{\sqrt{a^2+x^2}} = \operatorname{sen}\underline{h}^{-1}\frac{x}{a} + C$ arcsin$\underline{h}$

$\displaystyle\int e^{ax}\,dx = \frac{e^{ax}}{a} + C$

$\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}} = \operatorname{sen}^{-1}\frac{x}{a} + C$ arcsin

$\displaystyle\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$ arcsec

$\displaystyle\int \frac{dx}{\sqrt{x^2-a^2}} = \cos\underline{h}^{-1}\frac{x}{a} + C$ arccos$\underline{h}$

$\displaystyle\int x^n e^x\,dx = x^n e^x - n x^{n-1} e^x + n(n-1)x^{n-2}e^x - \dots + n!\,x e^x - n!\,e^x + C$

Números complejos: Euler

- $z = re^{i\theta}$ - $e^{i\theta} = (\cos\theta + i\sin\theta)$ ∴ $z = r(\cos\theta + i\sin\theta)$

De Moivre's: - $z^n = r^n e^{in\theta} = r^n[\cos(n\theta) + i\sin(n\theta)]$

Conversiones: - $a\cos\theta = ae^{i\theta}$ - $a\sin\theta = -ae^{i\theta}$

Funciones

$\mathcal{L}\{f(t)\} = \displaystyle\int_0^\infty e^{-st} f(t)\,dt$

Transformada de Laplace

$\mathcal{F}\{f(x)\} = \displaystyle\int_{-\infty}^{\infty} f(x)\, e^{i\alpha x}\,dx = F(\alpha)$

Transformada de Fourier

$\Gamma(n) = \displaystyle\int_0^\infty x^{n-1} e^{-x}\,dx = (n-1)!$

Función gamma

Funciones Hiperbólicas Inversas

$\boxed{\operatorname{sech}^{-1} x = \cosh^{-1}\frac{1}{x}}$ $\boxed{\operatorname{csch}^{-1} x = \sinh^{-1}\frac{1}{x}}$ $\boxed{\coth^{-1} x = \tanh^{-1}\frac{1}{x}}$

$\sinh^{-1}x = \ln(x+\sqrt{x^2+1})$	$-\infty < x < \infty$	$\operatorname{sech}^{-1}x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$	$0 < x \leq 1$
$\cosh^{-1}x = \ln(x+\sqrt{x^2-1})$	$< x \geq 1$	$\operatorname{csch}^{-1}x = \ln\left(\frac{1}{x}+\frac{\sqrt{1+x^2}}{\lvert x\rvert}\right)$	$x \neq 0$
$\tanh^{-1}x = \frac{1}{2}\ln\frac{1+x}{1-x}$	$\lvert x\rvert < 1$	$\coth^{-1}x = \frac{1}{2}\ln\frac{x+1}{x-1}$	$\lvert x\rvert > 1$

Derivadas e integrales de las funciones hiperbólicas inversas

$f(u)$	$\frac{df}{du}$
$\operatorname{senh}^{-1}u$	$\frac{1}{\sqrt{1+u^2}}\,du$
$\cosh^{-1}u$	$\frac{1}{\sqrt{u^2-1}}\,du$
$\tanh^{-1}u$	$\frac{1}{1-u^2}\,du$
$\coth^{-1}u$	$\frac{1}{1-u^2}\,du$
$\operatorname{sech}^{-1}u$	$\frac{1}{u\sqrt{1-u^2}}\,du$
$\operatorname{csch}^{-1}u$	$\frac{1}{\lvert u\rvert\sqrt{1+u^2}}\,du$

$$\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\frac{u}{a} + C$$

*

$$\int \frac{du}{a^2-u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\frac{u}{a}+C & \text{si } \lvert u\rvert < 1 \\ \frac{1}{a}\coth^{-1}\frac{u}{a}+C & \text{si } \lvert u\rvert > 1 \end{cases}$$

$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\frac{u}{a}+C$$

$$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\operatorname{csch}^{-1}\left|\frac{u}{a}\right|+C$$

Ejemplo!

$$\int \frac{2\,dx}{\sqrt{3+4x^2}} \qquad du$$

$u^2 = 4x^2 \Rightarrow u = 2x$ y $du = 2$. $a^2 = 3 \Rightarrow a = \sqrt{3}$

$$\int \frac{2\,dx}{\sqrt{3+4x^2}} = \int \frac{du}{\sqrt{a^2+u^2}} = \operatorname{senh}^{-1}\frac{u}{a} + C = \boxed{\operatorname{senh}^{-1}\frac{2x}{\sqrt{3}} + C}$$

* Ejemplo: $\int_{\frac{5}{4}}^{2} \frac{dx}{1-x^2}$: sea $a=1$ y $u=x$, $x>1$ $\forall x \in [\frac{5}{4}, 2]$. Por tanto,

$$\int_{\frac{5}{4}}^{2} \frac{dx}{1-x^2} = \coth^{-1}x\Big]_{\frac{5}{4}}^{2} = \coth^{-1}2 - \coth^{-1}\frac{5}{4} = \ln\sqrt{3} - \ln 3 = \ln\frac{\sqrt{3}}{3}$$

Usando el arcotangente hiperbólico:

$$\int_{\frac{5}{4}}^{2} \frac{dx}{1-x^2} = \tanh^{-1}x\Big]_{\frac{5}{4}}^{2} = \tanh^{-1}2 - \tanh^{-1}\frac{5}{4}$$

Pero $\tanh^{-1}2 \;\frac{1}{2}\ln\frac{1+2}{1-2} = \frac{1}{2}\ln -\frac{3}{1}$. Y $\ln(-3)$ no está definido.

- Si el intervalo de la integral contiene 1, la integral es divergente.

Sustitución trigonométrica

$$\int_0^{\ln 4} \frac{e^t\,dt}{\sqrt{e^{2t}+9}}$$

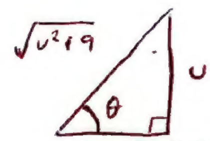
- Se computa la antiderivada. Se usa la sustitución $u^2 = e^{2t}$, $u = e^t$, y $du = e^t$.

$$\int \frac{e^t\,dt}{\sqrt{e^{2t}+9}} = \int \frac{du}{\sqrt{u^2+9}} \quad \leftarrow e^t = du$$

- Dado que u^2 y 9 son cuadrados perfectos, se usa una sustitución trigonométrica.
 Sea $a = 3$, $u = 3\tan\theta$, $du = 3\sec^2\theta$ y $\theta = \arctan\frac{u}{3}$:

$$\int \frac{du}{\sqrt{u^2+9}} = \int \frac{3\sec^2\theta\,d\theta}{\sqrt{9\tan^2\theta+9}} = \int \frac{3\sec^2\theta\,d\theta}{\sqrt{9(\tan^2\theta+1)}} = \int \frac{3\sec^2\theta\,d\theta}{\sqrt{9\sec^2\theta}} = \int \frac{\cancel{3}\sec^{\cancel{2}}\theta\,d\theta}{\cancel{3}\sec\theta} = \int \sec\theta\,d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$



$a = 3$ $(a^2 = 9)$

$u = a\tan\theta = 3\tan\theta$

$du = 3\sec^2\theta$

$\sec\theta = \frac{\sqrt{u^2+9}}{3}$

$\tan\theta = \frac{u}{3}$

$\theta = \arctan\left(\frac{u}{3}\right)$

- Se deshacen las sustituciones:

$$\ln|\sec\theta + \tan\theta| + C = \ln\left|\frac{\sqrt{u^2+9}}{3} + \frac{u}{3}\right| + C = \boxed{\ln\left|\frac{\sqrt{e^{2t}+9}+e^t}{3}\right| + C}$$

- Se computa la integral usando la antiderivada

$$\int_0^{\ln 4} \frac{e^t\,dt}{\sqrt{e^{2t}+9}} = \ln\left|\frac{\sqrt{e^{2t}+9}+e^t}{3}\right|\Bigg]_0^{\ln 4} = \ln\left(\frac{\sqrt{25}+4}{3}\right) - \ln\left(\frac{\sqrt{10}+1}{3}\right)$$

$e^{2\ln 4} = 4^2 = 16$, $e^{\ln 4} = 4$

$$= \ln\left(\frac{9}{3}\right) - \ln\left(\frac{\sqrt{10}+1}{3}\right) = \ln 9 - \cancel{\ln 3} - \left(\ln(\sqrt{10}+1) - \cancel{\ln 3}\right) = \boxed{\ln 9 - \ln(\sqrt{10}+1)} \quad 0'771162138$$

$$\int \frac{x^2+1}{x^4+1}\,dx = \int \frac{x^2\left(1+\frac{1}{x^2}\right)}{x^2\left(x^2+\frac{1}{x^2}\right)}\,dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}\,dx$$

- Se usa la sustitución $u = x - \frac{1}{x}$, de modo que $u^2 = \left(x-\frac{1}{x}\right)^2 = \left(x^2+\frac{1}{x^2}\right) - 2$, y
 $du = 1 + \frac{1}{x^2}$. Y $a = \sqrt{2}$ $(a^2 = 2)$.

$$\int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}\,dx = \int \frac{du}{u^2+2} = \frac{1}{\sqrt{2}}\arctan\left(\frac{u}{\sqrt{2}}\right) = \boxed{\frac{1}{\sqrt{2}}\arctan\left(\frac{x}{\sqrt{2}} - \frac{1}{x\sqrt{2}}\right) + C}$$

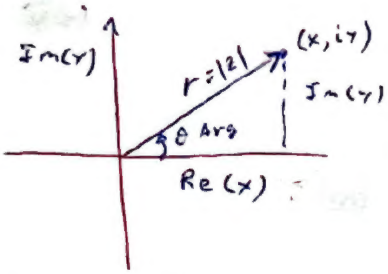
si $u^2 = x^2 + \frac{1}{x^2} - 2$, entonces $u^2 + 2 = x^2 + \frac{1}{x^2}$.

Complex number $i = \sqrt{-1}$

$Re + i\,Im$ or $Re + j\,Im$ $\qquad (5+8i) \rightarrow Re = x = 5,\ Im = y = 8$

$x = Re \qquad y = Im$

Argand Diagram



$\sin\theta = \frac{y}{r} = \frac{Im}{r} \qquad y = r\sin\theta$

$\cos\theta = \frac{x}{r} = \frac{Re}{r} \qquad x = r\cos\theta$

$Arg(z) = \theta = \arcsin\frac{y}{r} = \arcsin\frac{Im}{r} = \arccos\frac{x}{r} = \arccos\frac{Re}{r}$

Euler Formula

$z = re^{i\theta} \qquad z^n = r^n e^{in\theta} = r^n(\cos(n\theta) + i\sin(n\theta))$ De Moivre's

$e^{i\theta} = (\cos\theta + i\sin\theta)$

$z = r(\cos\theta + i\sin\theta)$

Equality

$a+bi = \boxed{c+di \Leftrightarrow a=c,\ b=d}$

Sum $(a\pm bi) \pm (c\pm di) = \boxed{(a\pm c) \pm (b\pm d)i}$

Product $(a+bi)(c+di) = \boxed{(ac-bd) + (ad+bc)i}$

Quotient $\frac{a+bi}{c+di} = \boxed{\frac{ac+bd}{a^2+b^2} - \frac{ad-bc}{a^2+b^2}i} \quad \boxed{\frac{a+bi}{c+di}\frac{(c-di)}{(c-di)}}$

Power $z^n = r^n e^{in\theta}$

Conversions

A=Constant

$\boxed{i=\sqrt{-1}}\ \boxed{i^2=-1} \quad \boxed{\sqrt{-4} = \sqrt{4}\sqrt{-1} = 2i} \quad \boxed{A\cos(\theta) = Ae^{i\theta}} \quad \boxed{A\sin(\theta) = -Ae^{i\theta}}$

$z = -7+24i \qquad |z| = \sqrt{49+576} = 25$

$z = -25(\cos\theta - i\sin\theta) \qquad arg(z) = \arcsin\frac{24}{25} = 1'287002$

using $z = re^{i\theta}$

$r = e^{i\theta} + e^{-i\theta}$

Equation

$(3+4i)^2 - 2(x - iy) = x + iy$

$(3+4i)^2 = (3+4i)(3+4i) = (9-16) + (12+12)i = (-7+24i)$

$-7+24i - 2x + 2iy = x + iy \rightarrow x = Re,\ y = Im$

$x = -7-2x \qquad y = 24 - 2y$

$\boxed{x = -\frac{7}{3}} \qquad \boxed{y = -24}$

Example

$$x = \frac{5 \pm \sqrt{-4}}{2} \rightarrow x = \frac{5 \pm i\sqrt{4}}{2} \rightarrow x = \frac{5 \pm 2i}{2} = \frac{1}{2}(5 \pm 2i) = \boxed{\left(\frac{5}{2} \pm i\right)}$$

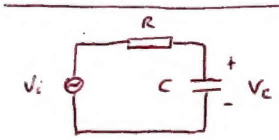
$$\boxed{z = \frac{5}{2} + i \quad \bar{z} = \frac{5}{2} - i} \rightarrow |z| = \sqrt{\left(\frac{5}{2}\right)^2 + 1^2} = \sqrt{\frac{25}{4} + \frac{4}{4}} = \sqrt{\frac{29}{4}} = \frac{1}{2}\sqrt{29}$$

$$r = |z| = \frac{1}{2}\sqrt{29} \rightarrow z = re^{i\theta} \rightarrow e^{i\theta} = \cos\theta + i\sin\theta \rightarrow z = \frac{1}{2}\sqrt{29}(\cos\theta + i\sin\theta)$$

$$\arg(z) = \theta \rightarrow \theta = \arcsin\frac{y}{r} = \arccos\frac{x}{r} = \arctan\frac{y}{x} \rightarrow \theta = \arctan\frac{2}{5}$$

$$\boxed{z = \frac{1}{2}\sqrt{29}\left(\cos\left(\arctan\frac{2}{5}\right) + i\sin\left(\arctan\frac{2}{5}\right)\right)} \quad \boxed{z = \frac{1}{2}\sqrt{29}\, e^{i\arctan\left(\frac{2}{5}\right)}}$$

$$\boxed{\bar{z} = \frac{1}{2}\sqrt{29}\left(\cos\left(-\arctan\frac{2}{5}\right) + i\sin\left(-\arctan\frac{2}{5}\right)\right)} \quad \boxed{\bar{z} = \frac{1}{2}\sqrt{29}\, e^{-i\arctan\left(\frac{2}{5}\right)}}$$



Find V_c

A: Some constant (Real)
j: $\sqrt{-1}$ (complex part)
ω: Phase

$$\boxed{V_i = V_c + RC\frac{dV_c}{dt}} \qquad \frac{dV_c}{dt} = j\omega V_c$$

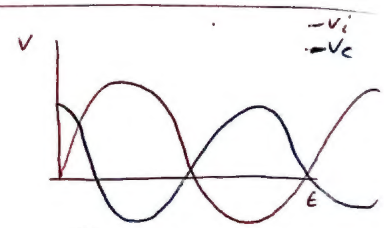
$$\boxed{V_c = A\cos(\omega t + \theta)} \qquad A\cos(\theta) = Ae^{j(\theta)}$$

$$V_c = Ae^{j(\omega t + \theta)}$$

$V_i = V_c + RC(j\omega V_c)$ Subst. $\frac{dV_c}{dt}$

$V_i = V_c(1 + j\omega RC)$ Factor V_c

$\boxed{V_c = \frac{1}{1 + j\omega RC} V_i}$ Divide both by $1 + j\omega RC$



ω Phase: $\cos(t+90)$

Capacitor eq.

$$\boxed{C\frac{dV}{dt} = I} \qquad \boxed{V = Ae^{j(\omega t + \theta)}} \quad I = j\omega C\underbrace{Ae^{j(\omega t + \theta)}}_{V} \qquad I = j\omega CV \qquad \frac{V}{I} = \frac{1}{j\omega C}$$

$\boxed{V = A\cos(\omega t + \theta)}$

(capacitor impedance)

Actual voltage is the real component.

Asuming $\omega = R = c = 1$

$$V_c = \frac{1}{1+j}V_i \rightarrow V_c = \left(\frac{1}{2} - \frac{1}{2}j\right)V_i \rightarrow V_c = \frac{\sqrt{2}}{2}e^{j(-45^\circ)}V_i$$

$$V_c = \frac{\sqrt{2}}{2}e^{j(-45^\circ)} \cdot 3e^{j(t+10^\circ)} = \frac{3\sqrt{2}}{2}e^{j(t-35^\circ)} = \boxed{\frac{3\sqrt{2}}{2}\cos(t - 35^\circ)}$$ (Ignoring the imaginary part)

$$I = -\omega CA\sin(\omega t + \theta) = j\omega CAe^{j(\omega t + \theta)} = j\omega CV$$

Descomposición parcial de fracciones

8.5 - 471

$$\int \frac{x^2+4x+1}{(x-1)(x+1)(x+3)}\,dx$$

1. Sea $x-r$ un factor linear de $g(x)$. Supon que $(x-r)^m$ es la mayor potencia de $x-r$ que divide $g(x)$. Entonces, a éste factor, asignale la suma de las m fracciones parciales:

$$\frac{A_1}{(x-r)} + \frac{A_2}{(x-r)^2} + \dots + \frac{A_m}{(x-r)^m}$$

Haz ésto para los distintos factores lineales de $g(x)$.

$$\boxed{x^2+4x+1 = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+3}}$$

2. Sea x^2+px+q un factor irreducible cuadrático de $g(x)$ tal que x^2+px+q no tenga una raíz real. Supon que $(x^2+px+q)^n$ es la máxima potencia de éste factor que divide $g(x)$. A éste factor, asignale la suma de las n fracciones parciales:

$$\frac{B_1x+C_1}{(x^2+px+q)} + \frac{B_2x+C_2}{(x^2+px+q)^2} + \dots + \frac{B_nx+C_n}{(x^2+px+q)^n}$$

Haz ésto para los factores cuadráticos de $g(x)$.

$$\begin{aligned} &A(x+1)(x+3) + B(x-1)(x+3) + C(x-1)(x+1) = \\ = \; &Ax^2+4Ax+3A + Bx^2+2Bx-3B + Cx^2 - C = \\ = \; &(A+B+C)(x^2) + (4A+2B)(x) + (3A-3B-C) \end{aligned}$$

3. Establece la fracción original $\frac{f(x)}{g(x)}$ igual a la suma de éstas fracciones parciales. Despeja la ecuación resultante y reordena los términos en orden decreciente de las potencias de x.

4. Iguala los coeficientes de las correspondientes potencias de x y resuelve la ecuación resultante para los coeficientes indeterminados.

$A+B+C = 1$ coef. x^2

$4A+2B+0C = 4$ coef. x^1

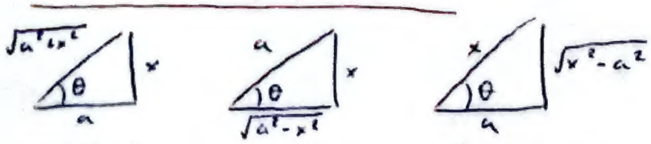
$3A-3B-C = 1$ coef. x^0

$$\text{rref}\left(\begin{bmatrix} 1 & 1 & 1 & 1 \\ 4 & 2 & 0 & 4 \\ 3 & -3 & -1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & \frac{3}{4} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{4} \end{bmatrix} \begin{bmatrix} x^2 \\ x^1 \\ x^0 \end{bmatrix}$$

$$A = \frac{3}{4},\; B = \frac{1}{2},\; C = -\frac{1}{4}$$

$$\int \frac{x^2+4x+1}{(x-1)(x+1)(x+3)}\,dx = \int \left[\frac{3}{4}\frac{1}{x-1} + \frac{1}{2}\frac{1}{x+1} + -\frac{1}{4}\frac{1}{x+3}\right] dx = \boxed{\frac{3}{4}\ln|x-1| + \frac{1}{2}\ln|x+1| - \frac{1}{4}\ln|x+3| + C}$$

Sustitución trigonométrica



$x = a\tan\theta$ $\quad$ $x = a\sin\theta$ $\quad$ $x = a\sec\theta$

$\sqrt{a^2+x^2} = a|\sec\theta|$ $\quad$ $\sqrt{a^2-x^2} = a|\cos\theta|$ $\quad$ $\sqrt{x^2-a^2} = a|\tan\theta|$

$$\int \frac{dx}{\sqrt{4+x^2}} = \int \frac{2\sec^2\theta\,d\theta}{\sqrt{4\sec^2\theta}} = \int \frac{\sec^2\theta\,d\theta}{|\sec\theta|} = \int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C = \ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C$$

$a = 2$

$x = 2\tan\theta \quad dx = 2\sec^2\theta\,d\theta \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$4+x^2 = 4+4\tan^2\theta = 4(1+\tan^2\theta) = 4\sec^2\theta$

Integración

$$\int_0^{\pi} e^{\cos\theta} \sin 2\theta \, d\theta = 2\int e^{\cos\theta} \sin\theta \cos\theta \, d\theta \qquad \sin 2\theta = 2\sin\theta\cos\theta$$

Sea $u = \cos\theta$, $du = -\sin\theta \, d\theta$, $-du = \sin\theta \, d\theta$

$$-2\int e^{u} \cdot u \, du$$

Es necesario una integración por partes. Cambiamos u por t y recalculamos los límites de la integración en base a t.

$t = u = \cos\theta$. $t(a) = t(0) = \cos 0 = 1$, $t(b) = t(\pi) = \cos\pi = -1$.

Siempre que haya un polinomio, u es el polinomio.

$$-2\int_1^{-1} e^{t} \cdot t \, dt = 2\int_{-1}^{1} e^{t} \cdot t \, dt$$

Sea $u = t$, $du = dt$, y sea $dv = e^{t}$, $v = \int e^{t} dt = e^{t}$:

$$2\left[te^{t}\Big]_{-1}^{1} - \int e^{t} dt \right] = 2\left[te^{t} - e^{t} \right]_{-1}^{1} = 2\left[(\cancel{1e^{1}} - \cancel{e^{1}}) - (-1e^{-1} - e^{-1}) \right] =$$

$$= 2\left(\frac{1}{e} + \frac{1}{e} \right) = \boxed{\frac{4}{e}}$$

Movimiento ideal de proyectiles

$$\boxed{r = (v_0 \cos\alpha) t \, i + \left((v_0 \sin\alpha) t - \frac{1}{2} g t^2 \right) j}$$

$$r = (x_0 + (v_0 \cos\alpha) t) i + (y_0 + (v_0 \sin\alpha) t - \frac{1}{2} g t^2) j$$

r: Función vectorial

v_0: Velocidad inicial (m/s)

t: Parámetro de tiempo (s)

g: Constante de gravedad (En la tierra $\approx 9'8$ m/s)

Altura máxima: $\dfrac{(v_0 \sin\alpha)^2}{2g}$

Tiempo de vuelo: $\dfrac{2 v_0 \sin\alpha}{g}$

Rango: $\dfrac{v_0^2}{g} \sin 2\alpha$